

**KENDRIYA VIDYALAYA ARMY CANTT. PANGODE**  
**MONTHLY TEST - AUGUST 2015**  
**MATHEMATICS**

Class XII

Time: 90 min

Max. Mark: 50

**General Instructions**

1. All questions are compulsory
2. The question paper consists of 15 questions divided into 3 sections A, B and C.
3. Section A comprises of 4 questions of one mark each, Section B comprises of 10 questions of four marks each and Section C comprises of 1 question of six marks.
4. Use of calculator is not permitted

**SECTION – A**

1. Find the point of discontinuity of the function  $f(x) = [x]$  in  $-3 < x < 5$ , where  $[x]$  is the greatest integer less than or equal to  $x$ .
2. If  $y = x^x$  then find  $\frac{dy}{dx}$
3. The cost function of a firm is given by  $C(x) = 300x - 10x^2 + \frac{1}{3}x^3$ . Find the marginal cost when 5 units are produced.
4. A balloon, which always remains spherical, has a variable radius. Find the rate at which its volume is increasing with the radius when later is 10 cm.

**SECTION – B**

5. For what value of  $\lambda$ , the function defined by  $f(x) = \begin{cases} \lambda(x^2 + 2) & \text{if } x \leq 0 \\ 4x + 6 & \text{if } x > 0 \end{cases}$  is continuous at  $x = 0$ ? Hence check the differentiability of  $f(x)$  at  $x = 0$
6. If  $y = (\sin x)^x + x^{\tan x}$  find  $\frac{dy}{dx}$
7. Verify Rolles theorem for  $f(x) = x^3 - 3x^2 + 2x$  in  $[0, 2]$
8.  $x = ae^b(\sin t + \cos t)$  and  $y = ae^b(\sin t - \cos t)$  prove that  $\frac{dy}{dx} = \frac{x+y}{x-y}$
9. If  $y = \sqrt{x+1} - \sqrt{x-1}$  then prove that  $(x^2 - 1)y_2 + xy_1 - \frac{1}{4}y = 0$

10. Differentiate  $\tan^{-1}\left(\frac{\sqrt{1+a^2x^2}-1}{ax}\right)$  with respect to  $\tan^{-1} x$ .

11. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle is  $\tan^{-1}\left(\frac{1}{2}\right)$ . Water is poured into it at a constant rate of 5 cubic meter per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10 m.

12. If  $x^y \cdot y^x = 1$  find  $\frac{dy}{dx}$

13. Differentiate  $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$  with respect to  $x$ .

14. If  $\cos y = x \cdot \cos(a + y)$  prove that  $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$

### SECTION – C

15. Find the intervals in which the function  $f(x) = x^3 - 12x^2 + 36x + 17$  is increasing or decreasing.